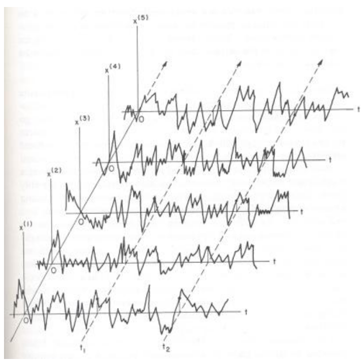
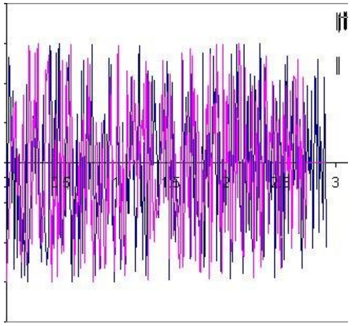
Twin Cities ANSYS® User Meeting

June 2011

Random Vibration





… within Epsilon

Cummins PowerGen

... within Epsilon

- Mechanical Engineer/ Experimental Mechanics
  - Cummins PowerGen/ Fridley, MN
    - PGBU Applied Technology Dept/
    - Applies engineering and problem solving skills to the design and development of generator sets and related systems in the area of Mech Dynamics and Fatigue evaluation.
  - Requirements: MS in Mech Eng or similar degree.
    - Strong analytical skills in Mech Dynamics with ability and experience working in the freq and time domain.
    - Exp in strain measurement and analysis.
    - Strong understanding in fatigue life evaluation including stress and strain life approach. Familiarity with eng tools,ANSYS,LMS,MATLAB and Ncode

For more info please contact John Garrigues/Sr Rec/Cummins

John.garrigues@cummins.com 763-528-7099

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Random Vibration

1. The Power Spectral Density (PSD) Curve
2. Random Vibration in Design
3. ANSYS implementation
   – APDL template
   – Workbench
4. Underlying Theory: Spectrum Response
   – Chris Wright

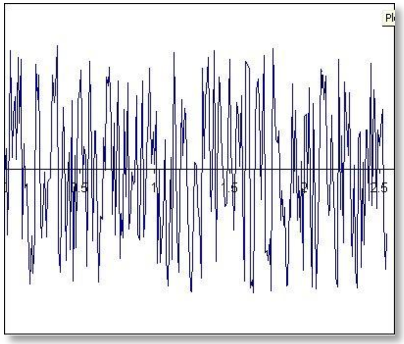
Credits

- Parts 1, 2, 3 of this presentation is adapted from PADT's Dynamics training course.
  - Excellent course written by Alex Grishin
  - PADT supplies custom and standard ANSYS training
  - See their course schedules and locations online
    - Contact Ted Harris
    - On-site training may be available

- Part 4 is Developed by Chris Wright

  chrisw@skypoint.com

What is random vibration analysis?



- A linear, mode-superposition technique for calculating a structure's response to a vibration load whose amplitude and frequency varies randomly with time.
- Meant for loads that produce unpredictable (random) time histories but nevertheless have predictable characteristics over time (frequency domain).
- Mil-Std-810 is common requirement in Aerospace
  - Some use static equivalent (e.g. Miles Equations)
  - Assumes single DOF! Can be weak approximation.
  - Not always conservative!
  - Non-linearities are numerically possible
    but doubtfully appropriate (potential abuse)

?

$$G_{RMS} = \sqrt{\frac{\pi}{2} f_n Q [ASD_{input}]}$$

A few other Applications:

- Aircraft electronic packaging
- Airframe parts under atmospheric loading
- Blast deflectors
- Laser guidance systems
- optical platform for telescopes
- Seismic loading of large structures
- Automobile suspension (bumpy road)
- Space launch vehicles and their payloads

The PSD Curve

Input:

- The structure's natural frequencies and mode shapes (so a modal extraction run must be performed first)
- The PSD curve (explained next)

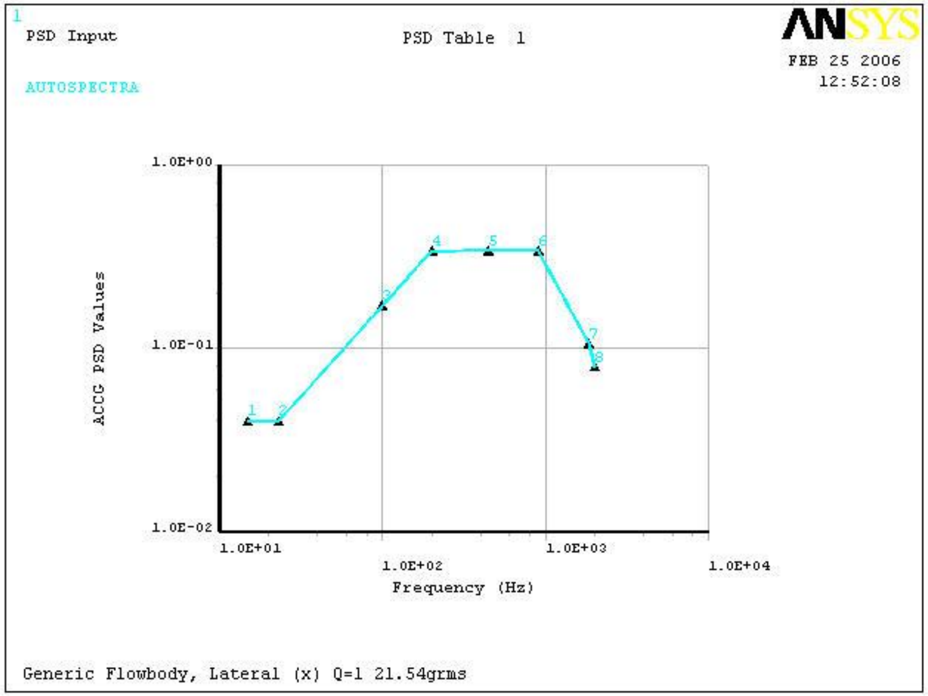
Output:

- 1σ displacements and stresses that can be used for fatigue life prediction (General Postprocessor).
- Response PSD curves that show the frequency content of any output quantity (RPSD – in the Time-History Postprocessor) at a given node.

The PSD Curve: An Example

... within Epsilon

- The frequency domain representation we use is the *power spectral density (PSD).* Transmission of this vibration through the structure is calculated by ANSYS.



Freq. (Hz)	G^2/Hz
15	0.04
23	0.04
100	0.1712
200	0.3405
450	0.3405
900	0.3405
1850	0.1056
2000	0.0794

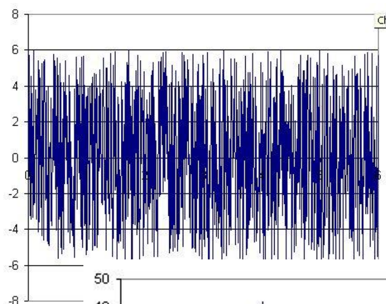
The PSD Curve: More Examples

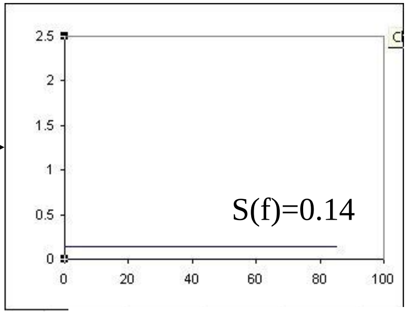
... within Epsilon

Below are three time signals (on the left), along with their PSD (on the right):

Equally distributed white noise

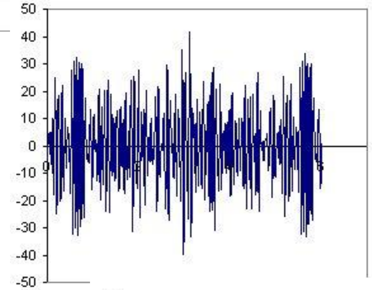
$\sigma^2 = 12$

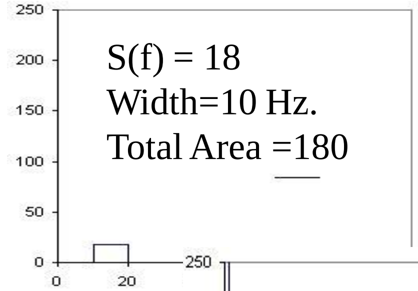




Gaussian sine wave superposition (10-20Hz)

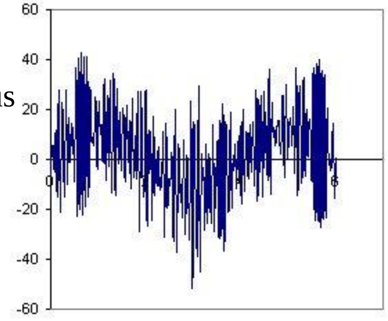
$\sigma^2 = 180$

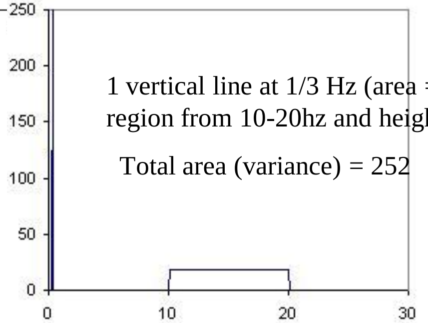




Sine-On-Random (previous curve + 1/3 Hz sine wave, amplitude 12)

$\sigma^2 = 252$





1 vertical line at 1/3 Hz (area = $12^2/2$). Square region from 10-20hz and height 18 (area = 180).

Total area (variance) = 252

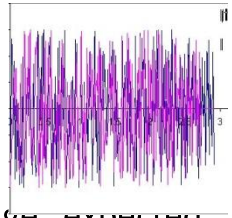
The PSD Curve

… within Epsilon

- A *PSD* records the *mean square* value of the excitation within an infinitesimal frequency interval as a function of frequency.
  - The area under a PSD curve is the **“variance”**, σ^2 of the response (square of the standard deviation, σ)
- Often called “power spectral density” in cases not dimensionally true
  - For *G^2/H curves* it’s actually power/mass
  - Acceleration Spectral Density (ASD) is the correct term for *G^2/Hz* curves

    (but is rarely used)
- What’s with the square term, anyway? *Such as G^2/Hz*
  - Need a positive sign for something that varies zero!
  - The RMS method is used
    - Can take sqrt of any particular frequency on the graph to obtain “average” expected acceleration input around that frequency.

$$\sqrt{x^2}$$

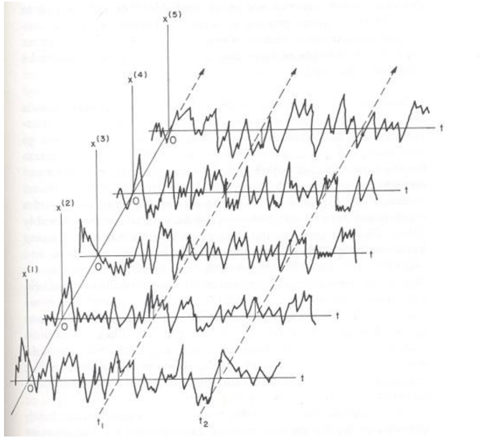


The PSD Curve

How to go from time varyting signal to PSD curve

The underlying math uses an *autoocorrelation function*, R of a signal V(t). It is defined as:

$$R(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} V(t)V(t+\tau)dt$$



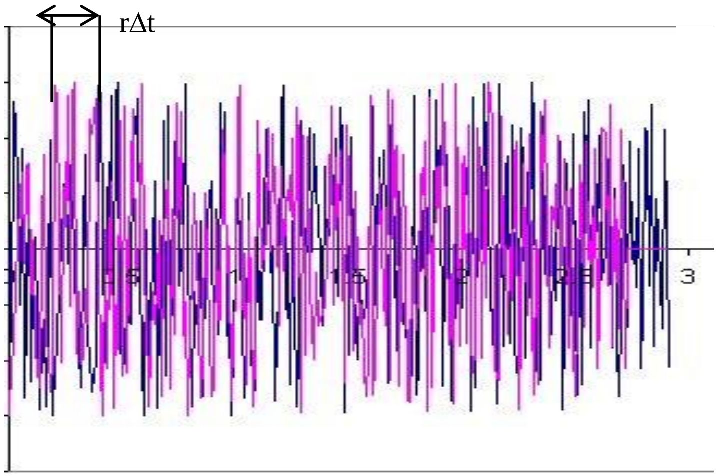
Reference: Random vibrations in mechanical systems by Crandall & Mark

Note if τ=0 this reduces to V^2 akin to simple RMS approach

The PSD Curve

... within Epsilon

This last definition of autocorrelation, R gives us yet another way of looking at it.



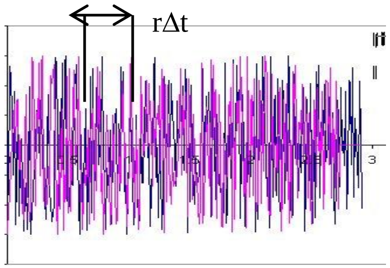
The violet curve is identical to the purple curve, but shifted by an amount rΔt. If Vn represents a point on the violet curve, then Vn+r represents a point on the purple curve to be multiplied to.

So, for different “shifts” (values of r) of the signal, we get different average sums of the products Vn*Vn+1 of the signal with itself!

The PSD Curve

... within Epsilon

So, for different "shifts" (values of r) of the signal, we get different average sums of the products Vn*Vn+1 of the signal with itself!



So now that we know that the autocorrelation function gives a measure of the average power of the signal per unit time. To get this information in terms of frequency components we use Fourier transformation!

$$S(f) = \int_{-\infty}^{\infty} R(\tau) e^{-2\pi f \tau} d\tau$$

This fact is known as the *Wiener-Khinchine Theorem*.

Easy as π!! (just kidding)

The PSD Curve: Freq. Distribution

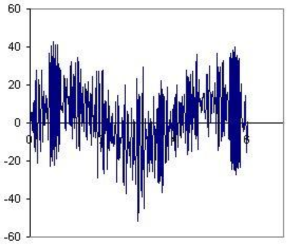
- Sine-On-Random

- Warning: In the“sine-on-random” example, the distribution is significantly non-Gaussian. Such a loading requires special treatment in order to assess nσ values.

- In some industries (definitely aerospace), a test specification may call for a random vibration environment superimposed on discreet- frequency constant harmonic vibration

- We can see how to extract a PSD from such a signal (in principle, anyway) with what we have already learned. As mentioned previously, a single harmonic excitation of infinite duration and constant amplitude V, phase φ, and frequency f has a PSD which is a vertical line at f. However, it has a finite area (variance) equal to $V^2/2$. It has an autocorrelation function equal to*

$$\frac{V^2}{2}\cos(2\pi ft-\varphi)$$



*Thomson, William T, Theory of Vibration With Applications, Third Edition, Chapter 13

The PSD Curve: Freq. Distribution

... within Epsilon

A single sine wave of constant frequency and phase, with amplitude A has the following distribution density:

$$p(x) = \frac{1}{\pi\sqrt{A^2 - x^2}} \quad |x| < A$$

And the following *cumulative probability**

$$P(x) = \frac{1}{2} + \frac{1}{\pi}\sin^{-1}\frac{x}{A} \quad |x| < A$$

Both of these measures are important in assessing a structure's potential damage or life. Probability distribution densities other than the Gaussian add a little more complexity. Adding a single sine wave to the random signal in the "sine-on-random" example completely changed its probability distribution. First, we'll focus on the Gaussian distribution.

*Once again, we'll just refer the student to Thomson, chapter 13

The PSD Curve: Freq. Distribution

For typical gaussian distribution, if a 1σ stress value is plotted in ANSYS, we immediately know that 68 percent of the time, the absolute value of this fluctuating quantity will be less than the plotted value. Multiplying this value by two gives a 95th percentile stress, and so on.

Also, for any distribution, an average frequency f^+ can be found for a quantity V(t) according to:

$$f^+ = \frac{\sigma_V}{2\pi\dot{\sigma}_V}$$

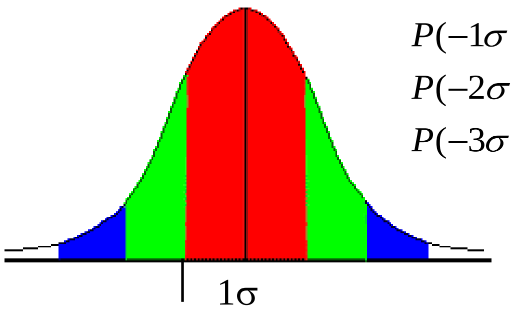
This quantity, along with the percentile stress can be used together with a material S-N curve to estimate the time, T_f to failure*

*For more details, see Newland, D.E., An Introduction to Random Vibrations and Spectral Analysis, Longman Group Ltd, London 1975

PSD – As a Design Criteria

- For stress results, one uses a probabilistic result based on **σ**
  - 3σ is a typical requirement
    - Might be negotiable with customer
    - Sometimes military will allow 2.7, 2.8, etc. which has proven survivable for some applications.
  - Field experience in flown aircraft says 3**σ** is a little conservative
- Various approaches to fatigue
  - Assume average frequency (previous slide) $f^+ = \frac{\sigma_V}{2\pi\dot{\sigma}_V}$
  - Or Miner Rule the cycles together
    - Assumes normal distribution with stresses scaled from 1 **σ** results.

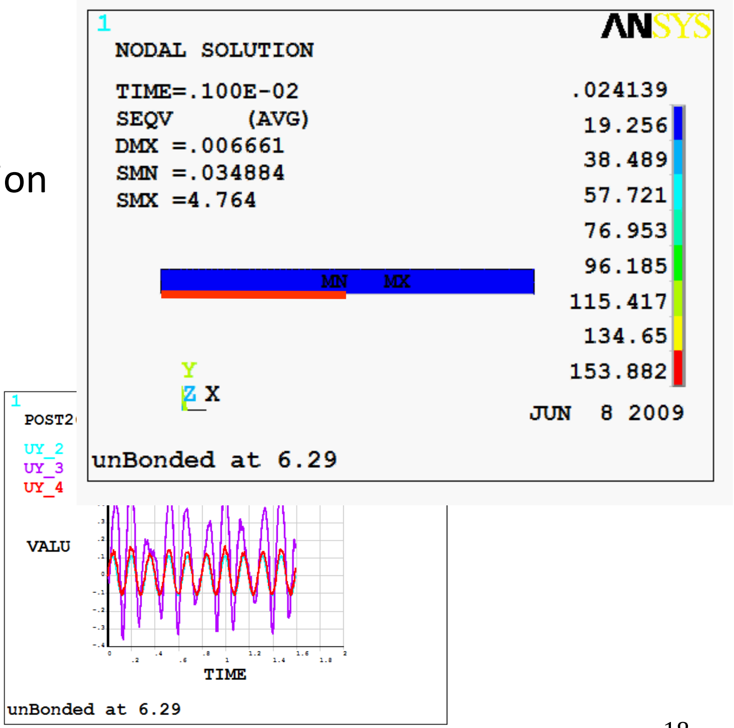
$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$



$$P(-1\sigma \le x(t) \le 1\sigma) = 0.68$$
$$P(-2\sigma \le x(t) \le 2\sigma) = 0.95$$
$$P(-3\sigma \le x(t) \le 3\sigma) = 0.997$$

PSD – Joints/Interfaces

- Damping of interfaces and joints is hard to quantify without correlated test data. Damping ratio of 0.02 is pretty easy to justify in most cases as conservative.
- Use same BC's in modal extraction as used in loading
- Use pinned bolt connections (held in UX, UY, UZ)
  - Not MX, MY, MZ
- For flanges/mating surfaces:
  - assume they are not in contact outside bolted region
  - Point connection, or
  - Annular region (1.5X bolt head diameter?)
- Above is well-supported by test experience
  - I even did a few simulation case studies



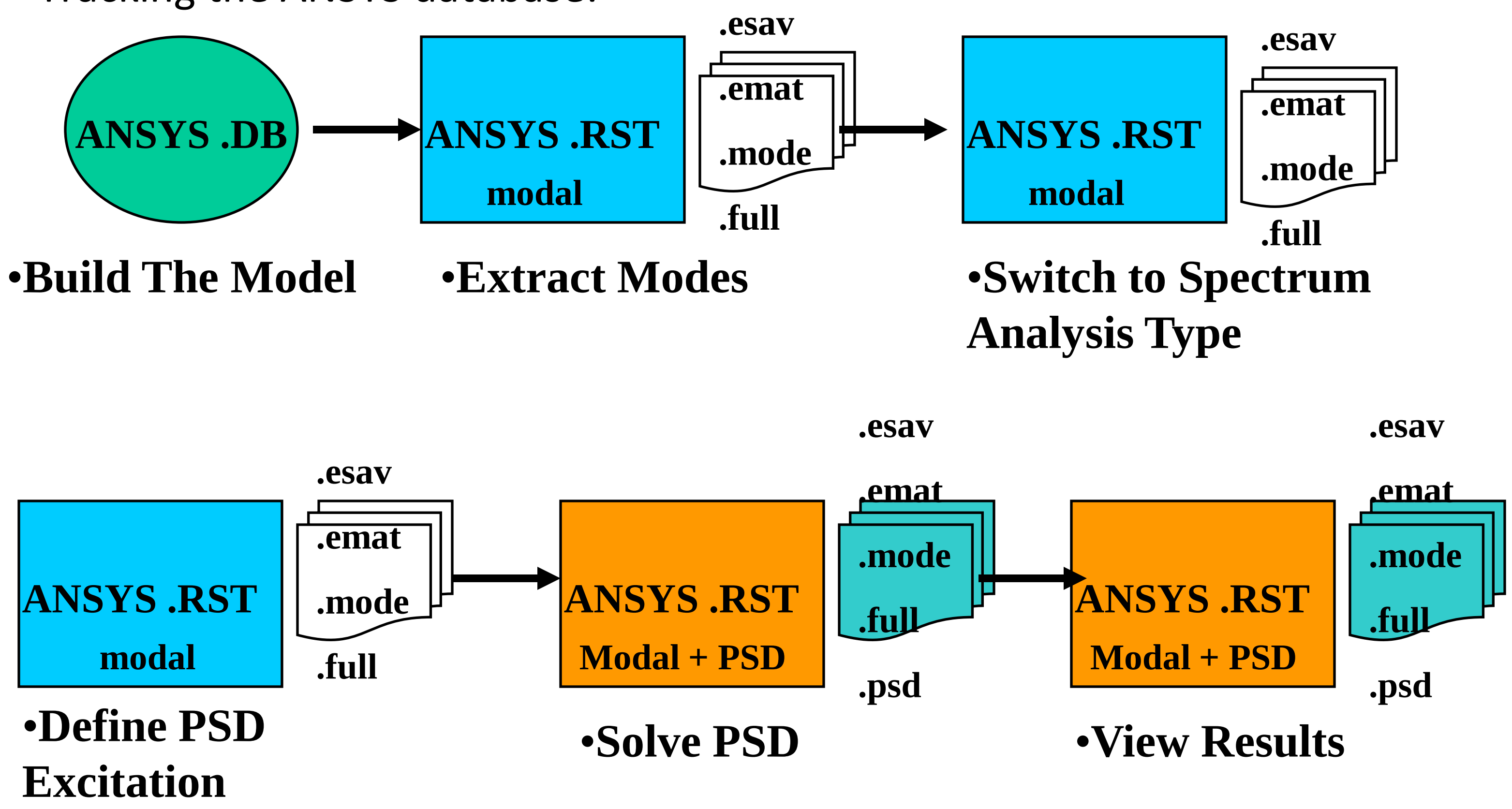
PSD – Reaction Forces

- Seems like someone always wants these…
  - One should question any non-statistical, non-fatigure related usage (e.g. it may not be the best way to size bolts
- PRRSOL (reaction solutions) are not useful!
- Must put a beam element connection to ground.
- Extract stresses/loads from the beams stress results.

As mentioned previously, ANSYS uses mode-superposition in the frequency-domain to solve the structure's response to a PSD load (slide 6). Because of this, a modal extraction must be performed before solving a random vibration problem (the solver needs the eigenvectors). The procedure is as follows:

1. **Build the model**
2. **Obtain the modal solution** **(pre-stress first, if necessary)**
3. **Switch to spectrum analysis type**
4. **Define and apply the PSD excitation** **(loads and boundary conditions, as well as damping are defined in this step)**
5. **Solve**
6. **Review results**

Tracking the ANSYS database:

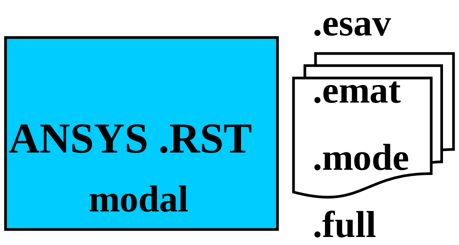


ANSYS .DB

- **Build The Model**

❖ In this step, the analyst builds the FE model as usual. Important things to remember: The Random Vibration solver is a linear solver. Any material nonlinearities will be ignored. However, the users CAN use the tangent stiffness matrix of the last converged geometrically nonlinear problem by updating the geometry from such a solution (UPCOORD). The user may also use the pre-stressed modal eigenvectors from a prior static run.

❖ The important thing to remember is that if a tangent stiffness matrix is to be used, a static (linear or nonlinear) solution, this analysis must be performed first, before modal extraction, and pre-stress effects must to turned ON.

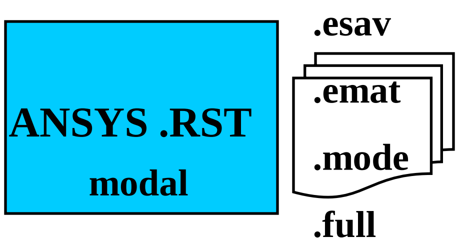


- **Extract Modes**

❖ Next, the user performs a modal analysis (include pre-stress effects if applicable).

❖ In this step, ANSYS generates the four files shown in the left, as well as the modal results file (.rst). The modal extraction run itself is saved as a load step in the .rst file. Each eigenvector is saved as a substep under this load step

❖ The other files store solver bookkeeping data such as element connectivity, matrices, and modal data



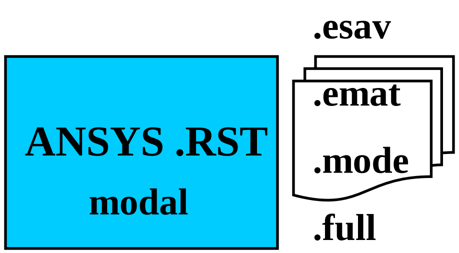
•Switch to Spectrum Analysis Type

❖ In this step, the user selects the PSD solver under "Spectrum", in the Solution option. Nothing happens to the database as this occurs:

NO CHANGE TO DATABASE

Random Vibration Analysis

ANSYS Procedure



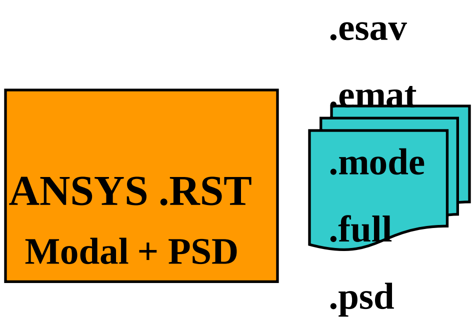
•Define PSD Excitation

❖ In this step, the user defines the PSD input(s), along with boundary conditions (if base loading is defined, no boundary conditions are specified), damping, and output controls. Still no change to the ANSYS database or file structure.

NO CHANGE TO DATABASE

Note:

The results file, as well as the four bookkeeping files must be as written after the modal run in order to perform the next step (solve). ANSYS will use all these files, and performing a PSD solution modifies these files and makes them unusable for subsequent PSD runs!



- **Solve PSD**

Note:

Because the results file gets appended to and bookkeeping files overwritten (and because ANSYS needs the unmodified files to do a PSD run), it is recommended that the user store the ANSYS modal database (results file + four bookkeeping file) to a separate folder for potential reuse

❖ In this step, the user solves the PSD transmission through the structure. The one-sigma solution is written to load steps 3, 4, and 5. Load step 2 stores the "quasi-static" solution (remember load step 1 stores the eigenvectors).

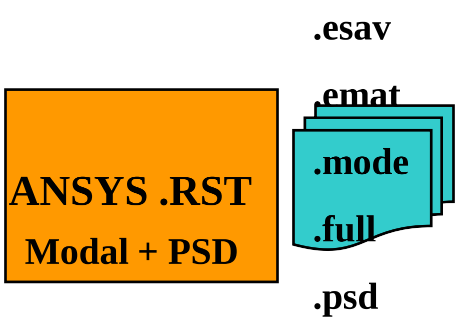
❖ Some of the bookkeeping files get modified, and a new file (.psd) gets written. This file stores participation factors, as well as other information needed to produce response PSD graphs in /post26.

❖ This step makes the ANSYS database unusable for subsequent PSD runs (you have to start all over again)

After "Solve", the results file (.rst) now contains:

- **Load Step 2**: quasi-static 1 sigma results
- **Load Step 3**: Displacement 1 sigma results
- **Load Step 4**: Velocity 1 sigma results
- **Load Step 5**: Acceleration 1 sigma results

Thus, for example, if the user wants to plot the model's 1σ stresses or displacements, he (she) will read results from load step 3. If velocities (these may be stress or strain velocities, not just the familiar variety) are desired, then results will be read from load step 4.



•View Results

In this step, the user may either:

❖ Enter the General Postprocessor and create one sigma model contour plots by first retrieving the desired load step, then plotting the desired one sigma quantity

❖ Enter the Time-History Postprocessor and create response PSD plots

MAPDL Template

- Perfect template in help manual

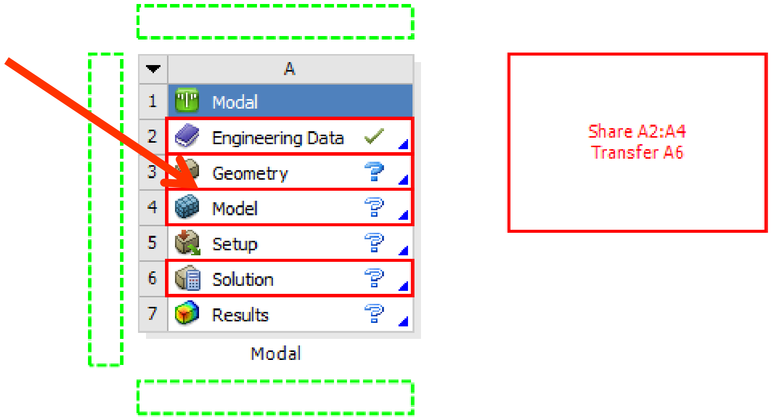
6.6.4 Sample Input

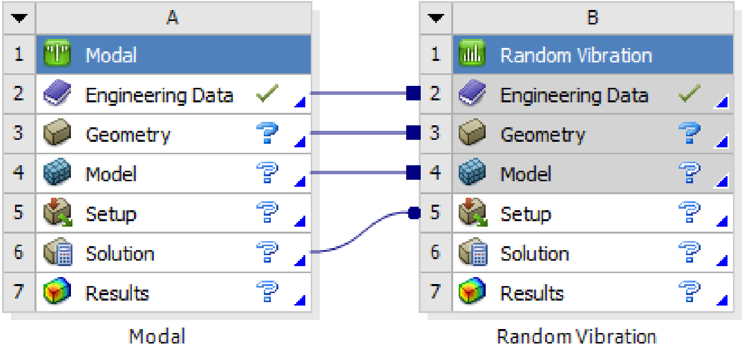
```
!
!  Obtain the Modal Solution
/SOLU                     ! Enter SOLUTION
ANTYPE,MODAL                 ! Modal analysis
MODOPT,LANB                  ! Block Lanczos method
MXPAND, ...               ! Number of modes to expand, ...
D, ...                  ! Constraints
SAVE
SOLVE                      ! Initiates solution
FINISH

!  Obtain the Spectrum Solution
/SOLU! Reenter SOLUTION
ANTYPE,SPECTR                ! Spectrum analysis
SPOPT,PSD, ...             ! Power Spectral Density; No. of modes;
PSDUNIT, ...               ! Type of spectrum
PSDFRQ, ...                ! Frequency pts. (spectrum values vs. frequency tables)
PSDVAL, ...                ! Spectrum values
DMPRAT, ...                ! Damping ratio
D,0                     ! Base excitation
PFACT, ...                 ! Calculate participation factors
PSDRES, ...                ! Output controls
SAVE
SOLVE
```

```
!  Combine modes using PSD method
/SOLU                          ! Re-enter SOLUTION
ANTYPE,SPECTR                   ! Spectrum analysis
PSDCOM,SIGNIF,COMODE            ! PSD mode combinations w
                                !ith significance factor and
SOLVE
FINISH
```

Workbench Implementation

- Insert a modal, and then a Random Vibration Branch
  - Must Drag the Random Vib to the Solution Line (red arrow)

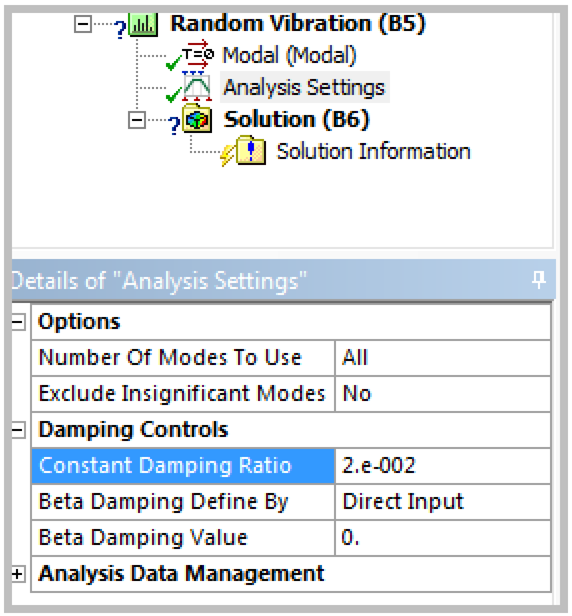


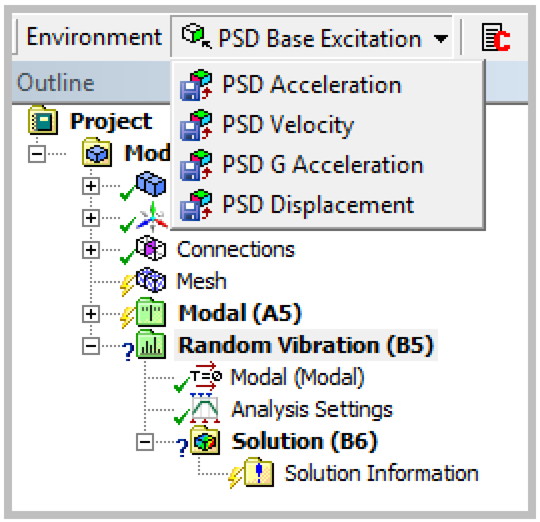


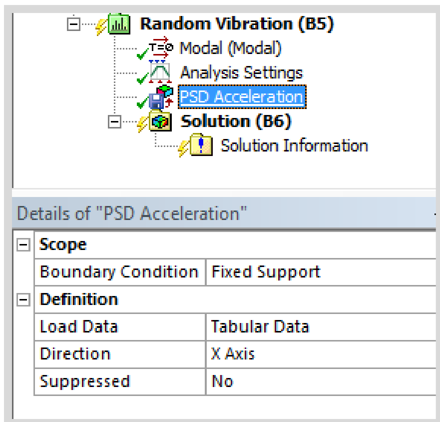
- In Modal Analysis branch, analysis settings, **specify to calc. stress/strains**!! (note that deflections-only is the default)

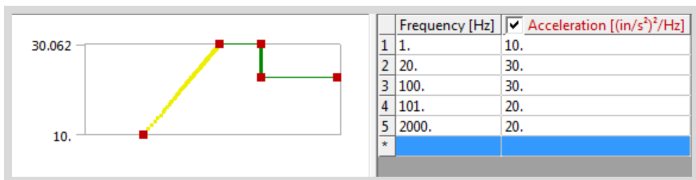
Workbench Implementation

- Define damping in analysis settings
- For applied loads, there is a special BC:
  - PSD Base Excitation
  - Specify direction and type (X, Y ,Z) and (Accel, G's, V, U)









Units of Measure

Quantity and Units of Measure

Quantity
- Physical entity
- Force, length, temperature
- Independent of units

Measure
- Pounds, Kilograms, Celsius
- Standard for comparison
- Always associated with units

US Customary Units

Length — inches (=2.54 cm)

Force — pound (=4.448 N)

Mass — lb-sec^2/in (= 5.7101E-3 Kg)

Density — lb-sec^2/in^4

Stress, pressure — lb/in^2 (=6895 Pa)

Force, Weight and Mass

Weight is the force exerted by gravity measured in pounds or Newtons

Mass is the quantity of matter measured in lb-sec^2/in or Kg

Force and mass are not independent

Weight = Mass x Acceleration of gravity
- [F] = [M][L][T^{-2}]
- [M] = [F][T^2][L^{-1}]

US Customary
- Force is fundamental
- Mass is derived

ISO/metric
- Mass is fundamental
- Force is derived

Chris wright's slides deleted -- see www.epsilonfea.com for full version.